

Nucleon Transversity Distribution from Azimuthal Spin Asymmetry in Pion Electroproduction

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Abstract

The azimuthal asymmetry observed by the HERMES collaboration in semi-inclusive pion production in deep inelastic scattering of unpolarized positron on the longitudinally polarized proton target, can provide information of the quark transversity distributions of the nucleon. We show that the quark transversity distributions predicted both by the light-cone quark-spectator-diquark model and by a pQCD inspired model can give consistent descriptions of the available HERMES data for the analyzing powers $A_{UL}^{\sin\phi}$ and $A_{UL}^{\sin 2\phi}$ for π^+ and π^- productions. We also show that the two models give similar predictions of $A_{UL}^{\sin\phi}$ for π^+ production, whereas they give very different predictions of $A_{UL}^{\sin\phi}$ for π^- production at large x . Further precision measurement of $A_{UL}^{\sin\phi}$ for π^- production can provide a decisive test of different models.

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Recently, the HERMES collaboration reported evidence for single-spin asymmetries for semi-inclusive pion production in deep inelastic scattering (DIS) of unpolarized positron beam on the longitudinally polarized proton target [1]. A significant spin asymmetry of the distribution in azimuthal angle ϕ of the pion relative to the lepton scattering plane is found. The lepton scattering plane is determined by the incident and scattered leptons, and the pion emitting plane is determined by the final detected pion and the virtual photon, with the virtual photon as the common axis of the two planes. The azimuthal angle ϕ is the angle between the two planes with the axis direction opposite to the virtual photon. Such azimuthal asymmetry offers a means to measure the nucleon transversity distributions, which are one of the three fundamental quark distributions of the nucleon. The other two are the unpolarized and helicity distributions, which are known with some precision both experimentally and theoretically. The quark transversity distribution is difficult to be measured, since it is not directly observable in inclusive DIS processes. Among the proposals to measure the quark transversity distributions, the azimuthal asymmetry in semi-inclusive hadron production has been considered [2, 3, 4, 5, 6], through the Collins effect [2] of non-zero production between a chiral-odd structure function and a T-odd fragmentation function. Indeed, there have been a number of studies [7, 8, 9, 10, 11, 12, 13] to show that the azimuthal asymmetries measured by HERMES can provide information concerning the quark transversity distributions of the nucleon.

The analyzing power measured by HERMES is defined as

$$A_{UL}^W = \frac{\int [d\phi] W(\phi) \{N^+(\phi) - N^-(\phi)\}}{\frac{1}{2} \int [d\phi] \{N^+(\phi) + N^-(\phi)\}}, \quad (1)$$

where UL denotes Unpolarized beam on a Longitudinally polarized target, $W(\phi) = \sin \phi$ or $\sin 2\phi$ is the weighting function for picking up the Collins effect, and $N^+(\phi)$ ($N^-(\phi)$) is the number of events for pion production as a function of ϕ when the target is positively (negatively) polarized. The analyzing powers for both π^+ and π^- are measured, and from the data there is clear evidence for the non-zero values of $A_{UL}^{\sin \phi}$ for π^+ , which indicate the azimuthal asymmetry. It has been found by Efremov *et al.* [8], based on a theoretical analysis presented in Refs. [4, 14], that the analyzing

powers $A_{UL}^{\sin\phi}$ and $A_{UL}^{\sin 2\phi}$ are, under a number of simplifying assumptions, proportional to the ratios

$$\frac{\sum_a e_a^2 \delta q^a(x) \langle \delta D^{a/\pi}(z)/z \rangle}{\sum_a e_a^2 q^a(x) \langle D^{a/\pi}(z) \rangle}, \quad (2)$$

and

$$\frac{x^2 \sum_a e_a^2 \left(\int_x^1 d\xi \delta q^a(\xi)/\xi^2 \right) \langle \delta D^{a/\pi}(z) \rangle}{\sum_a e_a^2 q^a(x) \langle D^{a/\pi}(z) \rangle}, \quad (3)$$

respectively. Here e_a is the charge of the quark with flavor a , $q^a(x)$ and $\delta q^a(x)$ are the quark unpolarized and transversity distributions of the nucleon target, and $D^{a/\pi}(z)$ and $\delta D^{a/\pi}(z)$ are the fragmentation functions to the pion π from an unpolarized and transversely polarized quark with flavor a . In fact the ratios taken from Ref. [4] involve higher twist functions and functions with intrinsic transverse momentum, and the latter functions are also related to twist three functions. In a next step all these functions are rewritten into twist two distribution and fragmentation functions using Wandzura-Wilczek type of relations, which are approximations. A further detailed theoretical analysis can be found in Ref. [12]. Therefore with the inputs of $D^{a/\pi}(z)$ and $\delta D^{a/\pi}(z)$ under some simplifying assumptions, we are able to get the quark transversity distributions $\delta q^a(x)$ from the measured analyzing powers. We will, following Ref. [8], consider only the contributions from the favored fragmentation functions $D^{a/\pi}$ and $\delta D^{a/\pi}$, i.e., $D(z) = D^{u/\pi^+}(z) = D^{\bar{d}/\pi^+}(z) = D^{d/\pi^-}(z) = D^{\bar{u}/\pi^-}(z)$ and similarly for $\delta D^{a/\pi}$. The average values for $\langle D^{a/\pi}(z) \rangle$, $\langle \delta D^{a/\pi}(z) \rangle$, $\langle \delta D^{a/\pi}(z)/z \rangle$, and the corresponding parameters are also chosen the same as Ref. [8]. Therefore we only need the quark momentum and transversity distributions to calculate Eqs. (2) and (3).

We now focus our attention on the quark transversity distributions of the nucleon. It is widely known that the bulk features of the quark momentum and helicity distributions of the nucleon can be well described by the quark-spectator-diquark model [15, 16, 17] and a pQCD based counting rule analysis [18, 19, 20]. Both models have their own advantages and played important roles in the investigation of various nucleon structure functions. However, there are still some unknowns concerning the sea content of the nucleon and the large x behaviors of valence quarks. For example, there are still some uncertainties concerning the flavor decomposition of the quark

helicity distributions at large x , especially for the less dominant d valence quark of the proton. There are two different theoretical predictions of the ratio $\Delta d(x)/d(x)$ at $x \rightarrow 1$: the pQCD based counting rule analysis [20] predicts $\Delta d(x)/d(x) \rightarrow 1$ whereas the SU(6) quark-spectator-diquark model [17] predicts $\Delta d(x)/d(x) \rightarrow -1/3$. There are still no precision experimental data which can provide a decisive test of the above two different predictions. In the following analysis we will show that the same discrepancy also exists concerning the quark transversity distributions for the d valence quark at large x . We know that the azimuthal spin asymmetry in π^- production of unpolarized lepton DIS scattering on the longitudinally polarized proton target is sensitive to the quark transversity of the valence d quark at large x , therefore a measurement of the azimuthal asymmetry of the π^- production at large x should be able to provide a decisive test of the different predictions.

The SU(6) quark-spectator-diquark model [15, 16, 17] starts from the three quark SU(6) quark model wavefunction of the baryon, and if any one of the quarks is probed, re-organize the other two quarks in terms of two quark wavefunctions with spin 0 or 1 (scalar and vector diquarks), i.e., the diquark serves as an effective particle which is called the spectator. Some non-perturbative effects such as gluon exchanges between the two spectator quarks or other non-perturbative gluon effects in the hadronic debris can be effectively taken into account by the mass of the diquark spectator. The mass difference between the scalar and vector diquarks has been shown to be important for producing consistency with experimental observations of the ratio $F_2^n(x)/F_2^p(x) = 1/4$ at $x \rightarrow 1$ found in the early experiments [15, 16], and also for predicting the polarized spin dependent structure functions of the proton and the neutron at large x [16, 17]. The light-cone SU(6) quark-spectator-diquark model [17] is a revised version of the same framework, by taking into account the Melosh-Wigner rotation effect [21, 22], in order to discuss the quark helicity and transversity distributions of the nucleon. More explicitly, the quark helicity and transversity distributions should be written as [21, 22]

$$\Delta q(x) = \int [d^2\mathbf{k}_\perp] M_q(x, \mathbf{k}_\perp) \Delta q_{QM}(x, \mathbf{k}_\perp); \quad (4)$$

$$\delta q(x) = \int [d^2\mathbf{k}_\perp] \hat{M}_q(x, \mathbf{k}_\perp) \Delta q_{QM}(x, \mathbf{k}_\perp), \quad (5)$$

where $\Delta q_{QM}(x, \mathbf{k}_\perp)$ is the quark spin distribution in the quark model, and $M_q(x, \mathbf{k}_\perp) = \frac{(k^+ + m)^2 - \mathbf{k}_\perp^2}{(k^+ + m)^2 + \mathbf{k}_\perp^2}$ and $\hat{M}_q(x, \mathbf{k}_\perp) = \frac{(k^+ + m)^2}{(k^+ + m)^2 + \mathbf{k}_\perp^2}$ are the corresponding Melosh-Wigner rotation factors due to the relativistic effect of the quark transversal motion. Some further detailed discussions can be found in Refs. [23, 24].

We need to point out that the quark-diquark model with simple wavefunctions can provide good relations between different quantities where the uncertainties in the model can be canceled between each other. It is impractical to expect a good description of the absolute magnitude and shape for a physical quantity. However, we may use some useful relations to connect the unmeasured quantities with the measured quantities. For example, we may use the following relation to connect the quark transversity distributions with the quark unpolarized distributions

$$\begin{aligned}\delta u_v(x) &= [u_v(x) - \tfrac{1}{2}d_v(x)]\hat{W}_S(x) - \tfrac{1}{6}d_v(x)\hat{W}_V(x); \\ \delta d_v(x) &= -\tfrac{1}{3}d_v(x)\hat{W}_V(x),\end{aligned}\tag{6}$$

in a similar way as was done for the quark helicity distributions [17]. We can use the valence quark momentum distributions $u_v(x)$ and $d_v(x)$ from one set of quark distribution parametrization as inputs to calculate the quark transversity distributions, with inputs of $\hat{W}_S(x)$ and $\hat{W}_V(x)$ from model calculation [23]. In this way we can make more reliable prediction for the absolute magnitude and shape of a physical quantity than directly from the model calculation.

We notice that the d quark in the proton is predicted to have a negative quark helicity distribution at $x \rightarrow 1$, and this feature is different from the pQCD counting rule prediction of “helicity retention”, which means that the helicity of a valence quark will match that of the parent hadron at large x . Explicitly, the quark helicity distributions of a hadron h have been shown to satisfy the counting rule [19],

$$q_h(x) \sim (1 - x)^p,\tag{7}$$

where $p = 2n - 1 + 2\Delta S_z$. Here n is the minimal number of the spectator quarks, and $\Delta S_z = |S_z^q - S_z^h| = 0$ or 1 for parallel or anti-parallel quark and hadron helicities, respectively [20]. Therefore the anti-parallel helicity quark distributions are suppressed by a relative factor $(1 - x)^2$, and consequently $\Delta q(x)/q(x) \rightarrow 1$ as $x \rightarrow 1$. Taking

only the leading term, we can write the quark helicity distributions of the valence quarks as

$$\begin{aligned} q_i^\uparrow(x) &= \frac{\tilde{A}_{q_i}}{B_3} x^{-\frac{1}{2}} (1-x)^3; \\ q_i^\downarrow(x) &= \frac{\tilde{C}_{q_i}}{B_5} x^{-\frac{1}{2}} (1-x)^5, \end{aligned} \quad (8)$$

where $\tilde{A}_q + \tilde{C}_q = N_q$ is the valence quark number for quark q , $B_n = B(1/2, n+1)$ is the β -function defined by $B(1-\alpha, n+1) = \int_0^1 x^{-\alpha} (1-x)^n dx$ for $\alpha = 1/2$, and $B_3 = 32/35$ and $B_5 = 512/693$. The application of the pQCD counting rule analysis to discuss the unpolarized and polarized structure functions of nucleons can be found in Ref. [20], and the extension to the Λ can be found in Refs. [25, 26].

The quark transversity distributions are closely related to the quark helicity distributions. Soffer's inequality [27] constrains the quark transversity distributions by the quark unpolarized and polarized distributions, and there also exists an approximate relation [23], which connects the quark transversity distributions to the quark helicity and spin distributions. Two sum rules [24], connecting the integrated quark transversities to some measured quantities and two model correction factors with limited uncertainties, have been also recently obtained. For example, if we assume the saturation of Soffer's inequality $2|\delta q(x)| \leq q(x) + \Delta q(x)$ [27], then we obtain $\delta q = \frac{1}{2} [q(x) + \Delta q(x)] = q^\uparrow(x)$, and this suggests that in general we may express $\delta q(x)$ in terms of $q^\uparrow(x)$ and $q^\downarrow(x)$. All these considerations indicate that it is convenient to parameterize the valence quark transversity distributions in a similar form as the helicity distributions. Therefore we use as a second model

$$\delta q(x) = \frac{\hat{A}_q}{B_3} x^{-\frac{1}{2}} (1-x)^3 - \frac{\hat{C}_q}{B_5} x^{-\frac{1}{2}} (1-x)^5, \quad (9)$$

which clearly satisfies Soffer's inequality. These quark transversity distributions are constrained by the values of $\delta Q = \int_0^1 \delta q(x) dx$ from the two sum rules in Ref. [24], and we also use $\hat{A}_q + \hat{C}_q = N_q$ as a constraint, just as in the case of the helicity distributions, in order to reduce the number of uncertain parameters. With the inputs of quark helicity sum $\Sigma = \Delta U + \Delta D + \Delta S \approx 0.3$, the Bjorken sum rule $\Gamma^p - \Gamma^n = \frac{1}{6}(\Delta U - \Delta D) = \frac{1}{6}g_A/g_V \approx 0.2$, both obtained in deep-inelastic lepton-nucleon scattering experiments [23, 24], and taking the two model correction factors both to be equal to 1 for the two sum rules of quark transversities [24], we obtain

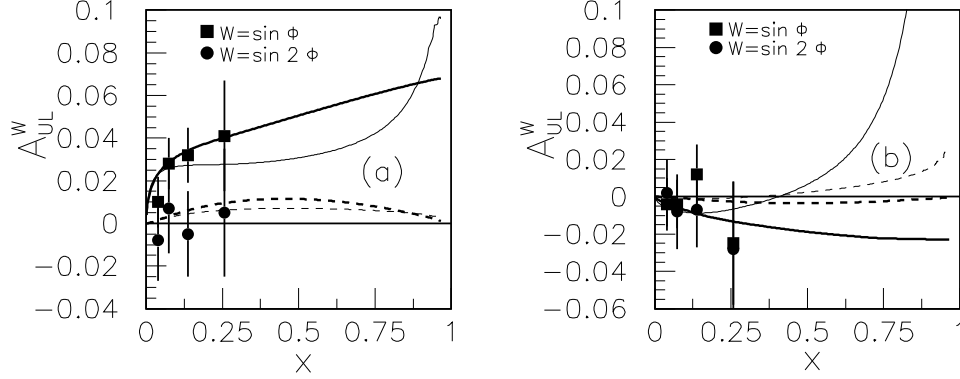


Figure 1: The analyzing powers $A_{UL}^{\sin \phi}$ and $A_{UL}^{\sin 2\phi}$ for (a) π^+ and (b) π^- semi-inclusive production in deep inelastic scattering of unpolarized positron on the longitudinally polarized proton target. The thick and thin curves correspond to the calculated results from the light-cone quark-diquark model and the pQCD inspired model respectively, with the solid and dashed curves corresponding to $A_{UL}^{\sin \phi}$ and $A_{UL}^{\sin 2\phi}$ respectively. The inputting quark distributions are from CTEQ5 set 1 parametrization [28] at $Q^2 = 4 \text{ GeV}^2$.

$\Delta U = 0.75$, $\Delta D = -0.45$, $\delta U = 1.04$, and $\delta D = -0.39$ for the proton, assuming $\Delta S = 0$. We may readjust the values when experimental constraints become available, or if we believe other models are more reasonable [24]. The parameters for quark distributions of the nucleons and the Λ can be found in Table 1. The ratios $\Delta q(x)/q(x)$ and $\delta q(x)/q(x)$ for the valence quarks of the proton are predicted to be 1 at $x \rightarrow 1$.

Table 1 The parameters for quark distributions of the proton in the pQCD inspired model

Baryon	q_1	q_2	\tilde{A}_{q_1}	\tilde{C}_{q_1}	\tilde{A}_{q_2}	\tilde{C}_{q_2}	\hat{A}_{q_1}	\hat{C}_{q_1}	\hat{A}_{q_2}	\hat{C}_{q_2}
p	u	d	1.375	0.625	0.275	0.725	1.52	0.48	0.305	0.695

In the denominators of Eqs. (2) and (3), there are contributions from both quark and antiquark fragmentations, and we should take them into account at small x . Using the CTEQ parametrization of quark distributions [28], we can calculate the contributions from quarks and antiquarks for the unpolarized quark distributions. We also use the values of $u_v(x)$ and $d_v(x)$ from the CTEQ parametrization as the inputs in Eq. (6) to calculate the quark transversity distributions of the valence quarks. This

is consistent with the calculation of the denominators, since we have the same inputs. In Fig. 1 we present the calculated $A_{UL}^{\sin\phi}$ and $A_{UL}^{\sin 2\phi}$ for both π^+ and π^- productions with only the valence quark distributions in the numerators of Eqs. (2) and (3). We can see from Fig. 1 that the calculated results are compatible with the HERMES data of the analyzing powers, and this is in agreement with existing interpretations of single-spin asymmetries as being associated with the valence quarks distributions [29, 30]. In Fig. 1 we also present the calculated results with the quark transversity distributions from the pQCD inspired model, and find that the results are also in compatible with the existing data.

We notice that the quark-diquark model and the pQCD inspired model give similar predictions of the analyzing power $A_{UL}^{\sin\phi}$ for π^+ production, whereas they give very different predictions of $A_{UL}^{\sin\phi}$ for π^- production. For π^+ production, the contributions of the quark transversity distributions are dominated by the u quarks from the fragmentation of $u \rightarrow \pi^+$. From Table 1 of Ref. [24], we find that the quark-diquark model and the two sum rules have similar predictions of δU , and also in both the quark-diquark model and the pQCD inspired model, the u quarks are totally positively polarized inside the proton at $x \rightarrow 1$. Therefore the analyzing power $A_{UL}^{\sin\phi}$ for the π^+ production is not sensitive to different models. However, this situation is quite different for π^- production. We find from Fig. 1(b) that the quark-diquark model and the pQCD inspired model have different predictions of $A_{UL}^{\sin\phi}$ for π^- production in the region of $x \geq 0.3$, right above the available experimental data. This can be easily understood since the quark-diquark model predicts $\delta d(x)/d(x) = -1/3$ at $x \rightarrow 1$, whereas the pQCD inspired model predicts $\delta d(x)/d(x) = 1$. For π^- production, the contributions of the quark transversity distributions are dominated by the d quark from the fragmentation of $d \rightarrow \pi^-$. Therefore $A_{UL}^{\sin\phi}$ for π^- production goes in opposite directions at large x in the two models. This prediction does not suffer from any model uncertainty which might change the calculated results quantitatively. Thus further precision measurement of $A_{UL}^{\sin\phi}$ for the π^- production by HERMES or other groups can provide a decisive test of the two different predictions. However, there should be some uncertainties in the quantitative predictions. The uncertainties for the quark-diquark model should be small, whereas the uncertainties for the pQCD

based model are comparatively large. Further constraints from more experimental data on the pQCD based model with higher order terms can improve the situation and increase our predictive power for the pQCD based analysis.

In conclusion, we showed in this paper that the azimuthal asymmetry observed by the HERMES collaboration in semi-inclusive pion production in deep inelastic scattering of unpolarized positron on the longitudinally polarized proton target, can provide information of the quark transversity distributions of the nucleon. We showed that the quark transversity distributions predicted both by the light-cone quark-spectator-diquark model and by a pQCD inspired model can give consistent descriptions of the available HERMES data of the analyzing powers $A_{UL}^{\sin\phi}$ and $A_{UL}^{\sin 2\phi}$ for both π^+ and π^- production. We also showed that the two models give similar predictions of $A_{UL}^{\sin\phi}$ for π^+ production, whereas they give very different predictions of $A_{UL}^{\sin\phi}$ for π^- production at large x . Further precision measurement of $A_{UL}^{\sin\phi}$ for π^- production can provide a decisive test of different models.

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